Chapter 5 Linear Momentum

The law of conservation of momentum states that if there are no outside forces acting on a system, the momentum of the system remains unchanged. This creates the basis for Newtonian Dynamics. Newton defined momentum as the product of mass and velocity.

$$p = mv$$

Because velocity is a vector quantity, it follows that momentum is too. When the velocity vector v is multiplied with the scalar quantity m, the vector p is in the direction of the velocity. This is the basis for linear momentum.

As it is for all vectors, oppositely directed momenta cancel. Recalling the addition of velocity vectors, if a train travels at 10m/s west, you can run off the end of a flatcar at 10m/s east and remain motionless with respect to the ground. It is this vector analysis that leads us to the conclusion that momentum acts similarly to velocity vectors. Linear momentum is relative, just as velocity.

Impulse & Change in Momentum

Considering Newton's second law:

$$F = ma$$

$$F = m(\Delta v / t)$$

$$F = (m \Delta v / t)$$

$$F = (\Delta p / t)$$

So Newton's Law states that the net force applied to an object equals the resulting change in its momentum through an interval of time Δt. For motion along a straight line, this becomes the formula for calculating impulse, or the change in momentum.

$$F\Delta t = \Delta p$$

A given impulse will produce a specific change in momentum no matter what the mass or speed of the recipient. A object initially at rest will accelerate in the direction of the applied force, acquiring momentum depending on the amount of time the force is applied to it: $\Delta p = F\Delta t$. This is what happens when a ball is kicked or a gun is fired. As long as the force is being applied to the object, momentum is increasing. Once the force stops acting on the object, the object will maintain momentum until acting upon by another force and causes another change in momentum. Ever wonder why sniper rifles are so long? The longer the object has the force applied to it, the more momentum it gains. In the case of the rifle, the long barrel leaves more time for the propelling force to act on the bullet, causing a greater momentum.

Conservation of Linear Momentum

As previously stated, the law of conservation of momentum states that when the resultant of all the external forces acting on a system is zero, the linear momentum of the system remains constant.

$$p_i = p_f$$

> *He set in motion in many different ways the parts of matter when He created them, and since He maintained them with the same behavior and with the same laws as He laid upon them in their creation, He conserves continually in this matter an equal quantity-of motion.*
>
> ~ *Descartes*

In the words of Descartes, the total momentum of the Universe is unchanged. In the words of the text, the total momentum of a system of interacting masses must remain unchanged, provided that there are no external forces applied. Simply put, conservation of momentum.

Collisions

A collision is defined by the transfer of momentum between objects in motion caused by their interaction. Let's say it again, in any case where there is no external forces present, total momentum must be conserved.

Inelastic Collisions

An inelastic collision is one where the final kinetic energy of the system is different from the initial kinetic energy. For example, a tennis ball (or any ball for that matter) which is dropped will bounce and compress against the ground and pop back into the air. If kinetic energy was completely conserved, the ball would bounce back to its initial position. However, energy is lost to heat, molecular shifting, sound, etc. and the ball only bounces about two-thirds of the way back up.

Completely inelastic collisions are just what they sound like, the objects are stuck together after they collide. In other words, all the kinetic energy is transformed (lost). A wad of clay against a wall is an example of a completely inelastic collision. All the energy you gave the clay was given to the wall. While completely inelastic collisions are extreme and generally not accepted as everyday events, it is in the best interest of learning to examine it.

If the linear momentum is to remain constant for a system of two objects:

$$p_{1i} + p_{2i} = p_{1f} + p_{2f} \; m_1 v_{1i}$$

$$+ m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

That is to say, final momentum of both objects equals the initial momentum brought in by both objects. Any problem relating to inelastic collisions will follow this mathematical example.

Elastic Collisions

An elastic collision is one where the final energy of the system is constant. This situation is accurately represented when a mass (m1) strikes another mass (m2) while the second mass is at rest.

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$$

The masses move off independently of each other when energy is conserved. When energy is conserved, one can formulate an equation for the transfer of energy relatively easily.

$$KE_{1i} + KE_{2i} = KE_{1f} + KE_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

The two final speeds are generally the unknowns in this equation. However, with a little algebra these two values can be calculated.

$$v_{1i} + v_{1f} = v_{2f}$$

$$v_{1i} = v_{2f} - v_{1f}$$

This means that the relative speeds of the two objects after the elastic collision is equal to the relative speeds of the objects before impact. This is true for all elastic collisions. Intuitively, this makes sense when you consider the fact that energy is conserved. If energy is proportional to velocity, and energy is conserved, shouldn't velocity be conserved as well?

Newton's cradle is a good example of an elastic collision of objects with equal mass. As the ball on one side slams into the line of balls with velocity v, it transfers its energy and momentum through a collision. Each ball in the line accepts the new energy and transfers it when a new collision. When the ball on the opposite side is not resisted by a force, it then flies out with energy and momentum equal to the original ball.

Collisions in Two Dimensions

Two dimensional collisions display the exact same concepts as linear collisions. However, as in most physical systems, components in the x and y-directions act independently. It is not surprising, but energy and momentum are conserved independently in each direction.

$$p_{ix} = p_{fx} \quad p_{iy} = p_{fy}$$

Of course, the inclusion of angles and trigonometry are necessary for these calculations. Interestingly, it is possible to prove that two objects undergoing a glancing elastic collision, in which one of them is initially at rest, always move off perpendicularly to each other.